## MATHEMATICAL SIMULATION <br> OF THE PROCESS OF PEAT IGNITION

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#### Abstract

Within the framework of a mathematical model of a porous reacting medium, the statement and numerical solution of the problem of occurrence of combustion (smouldering) of peat as a result of the action of a low fire seat are given. It is found that at moderate temperatures, $T \leq 700 \mathrm{~K}$, the smouldering of the initial reagent is limited by the processes of drying, pyrolysis, and by the reaction of oxidation of the carbon residue.


Introduction. It is known that underground (peat) fires cause a considerable detriment to the economics of various countries in the world. However, peat fires have been studied less adequately as compared to conventional forest fires because of the absence of information on the mechanism of ignition and propagation of combustion into a layer of peat and also because of the lack of reliable data on the thermophysical and thermokinetic coefficients of peat. In [1], based on the results of observation of actual peat fires in the Tomsk region, a general mathematical model of peat fires was suggested. Thereafter, the results of experimental investigations of peat fires were published [2], and based on [1] a series of works on their mathematical simulation was carried out [3-5] that confirm the physical foundations of the mathematical model of [1]. In [6] a refined second-generation mathematical model of peat fires was suggested, within the framework of which the two-temperature nature of the porous medium, particles of ashes, soot, smoke, and of free water and the influence of the multicomponent gas phase are take into account.

In the present work, based on the model of [6], with account for the experimental data of [7, 8], the occurrence of an underground fire is investigated in which the peat layer is ignited from above and the combustion front propagates inside the layer under various environmental conditions and different moisture contents of the peat.

1. Statement of the Problem. We will assume that fire on a pit $\log$ appears from a ground combustion seat, the action of which on a layer of peat is modeled by assignment of the temperature of the seat $T_{\mathrm{e}}$ and of heat- and mass transfer coefficients $\alpha_{\mathrm{e}}$ and $\beta_{\mathrm{e}}$. A one-temperature, one-dimensional nonstationary problem is considered; the $z$ axis is directed vertically downward, with the origin of coordinates being taken at the peat layer-atmosphere phase interface. According to [6], we assume that in the peat layer itself the following processes take place: evaporation of bound water (in contrast to [3-5], the process of water evaporation is considered equilibrium), an exothermal reaction of combustion of coke breeze, as well as homogeneous reactions of peat pyrolysis and combustion of carbon oxide. Therefore, we considered that in the process of ignition, peat is a multiphase medium consisting of dry organic matter, hygroscopic water stably bound with this organic matter [6] and the product of pyrolysis of organic matter (coke breeze), as well as of condensed and gaseous products of combustion. It was assumed that the gas phase in the peat layer consisted of six components: $\mathrm{CO}, \mathrm{H}_{2} \mathrm{O}, \mathrm{O}_{2}, \mathrm{CO}_{2}, \mathrm{CH}_{4}$, and $\mathrm{N}_{2}$. The influence of heat release due to the combustion of the $\mathrm{H}_{2}$ component is small by virtue of its insignificant initial concentration and low temperatures occurring in the process of smouldering of the permeable layer of peat. A layer of peat in which the initial volumetric fraction of the gas phase $\varphi_{5 \text { in }}\left(0.1<\varphi_{5 \text { in }}<0.2\right)$ is small as compared to the volumetric fractions of condensed phase was considered. Moreover, the processes of ignition and smouldering of peat were considered one-dimensional to simplify analysis. This mathematical model is a particular case of the model suggested in [6]. It is known that during combustion of peat, coke is formed as a result of the process of pyrolysis [1, 6] which thereafter smoulders in the course of the exothermic reaction of oxidation. The zone of pyrolysis is inside the front of combustion of the peat layer. The linear velocity of smouldering is determined from the equation

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$$
\begin{equation*}
T\left[s_{*}(t), t\right]=T_{*}, \tag{1}
\end{equation*}
$$

where $T_{*}$ is the temperature attainable at $z=s_{*}(t)$ and corresponding to the smouldering temperature of peat [2, 7]. Differentiating Eq. (1) with respect to time, we obtain an expression for the linear velocity of the smouldering of coke:

$$
\begin{equation*}
\omega=-\left.\left.\frac{\partial T}{\partial t}\right|_{z=s_{*}} \quad \frac{\partial T}{\partial z}\right|_{z=s_{*}} . \tag{2}
\end{equation*}
$$

The problem formulated mathematically above is reduced to the solution of the following system of equations $[6,9]:$

$$
\begin{align*}
& \frac{\partial \rho_{5} \varphi_{5}}{\partial t}+\frac{\partial \rho_{5} \varphi_{5} \nu}{\partial z}=Q,  \tag{3}\\
& \frac{\partial P}{\partial z}=-\frac{\mu}{\xi} v,  \tag{4}\\
& \sum_{i=1}^{4} c_{i \mathrm{~s}} \rho_{i \mathrm{~s}} \varphi_{i} \frac{\partial T}{\partial t}+\rho_{5} \varphi_{5} c_{p 5}\left(\frac{\partial T}{\partial t}+v \frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right)+q_{1} r_{1}+q_{2} r_{2}+q_{1 \mathrm{~s}} R_{1 \mathrm{~s}}-q_{2 \mathrm{~s}} R_{2 \mathrm{~s}}+q_{3 \mathrm{~s}} R_{3 \mathrm{~s}}+q_{4 \mathrm{~s}} R_{4 \mathrm{~s}},  \tag{5}\\
& \varphi_{5} \rho_{5}\left(\frac{\partial c_{\alpha}}{\partial t}+v \frac{\partial c_{\alpha}}{\partial z}\right)=\frac{\partial}{\partial z}\left(\rho_{5} \varphi_{5} D_{\alpha} \frac{\partial c_{\alpha}}{\partial z}\right)-c_{\alpha} Q+R_{\alpha}, \quad \alpha=1,2, \ldots, N-1,  \tag{6}\\
& \rho_{1 \mathrm{~s}} \frac{\partial \varphi_{1}}{\partial t}=-R_{1 \mathrm{~s}}, \quad \rho_{2 \mathrm{~s}} \frac{\partial \varphi_{2}}{\partial t}=-R_{2 \mathrm{~s}},  \tag{7}\\
& \rho_{3 \mathrm{~s}} \frac{\partial \varphi_{3}}{\partial t}=\alpha_{\mathrm{c}} R_{1 \mathrm{~s}}-R_{3 \mathrm{~s}}-\alpha_{4} R_{3 \mathrm{~s}}, \quad \rho_{4 \mathrm{~s}} \frac{\partial \varphi_{4}}{\partial t}=\alpha_{4} R_{3 \mathrm{~s}},  \tag{8}\\
& \sum_{\alpha=1}^{N} c_{\alpha}=1, \quad \varphi_{5}=1-\sum_{i=1}^{4} \varphi_{i}, \quad P=\frac{\rho_{5} R T}{M}, \quad M^{-1}=\sum_{\alpha=1}^{N} \frac{c_{\alpha}}{M_{\alpha}} .
\end{align*}
$$

In order to solve the system of equations (3)-(8) the following initial and boundary conditions are needed:

$$
\begin{gather*}
\left.T\right|_{t=0}=T_{\text {in }},\left.\quad \rho_{5}\right|_{t=0}=\rho_{5 \text { in }},\left.\quad c_{\alpha}\right|_{t=0}=c_{\alpha \text { in }}, \quad \alpha=1,2, \ldots, N-1,  \tag{9}\\
\left.\varphi_{i}\right|_{i=0}=\varphi_{i \text { in }}, \quad i=1, \ldots, 4, \\
\alpha_{\mathrm{e}}\left(T_{\mathrm{e}}-T\right)=\left.\lambda \frac{\partial T}{\partial z}\right|_{z=0},\left.\quad T\right|_{z \rightarrow \infty}=T_{\mathrm{in}} . \tag{10}
\end{gather*}
$$

Using the analogy between the processes of heat and mass transfer [10] $\left(\beta_{\mathrm{e}}=\alpha_{\mathrm{e}} / c_{p 5}\right)$, we have the boundary conditions

$$
\begin{gather*}
\beta_{\mathrm{e}}\left(c_{\alpha \mathrm{e}}-\left.c_{\alpha}\right|_{z=0}\right)=\left.\varphi_{5} \rho_{5} D_{\alpha} \frac{\partial c_{\alpha}}{\partial z}\right|_{z=0},\left.P\right|_{z=0}=P_{\mathrm{e}}  \tag{11}\\
\left.c_{\alpha}\right|_{z \rightarrow \infty}=c_{\alpha \text { in }}, \quad v=0, \quad \alpha=1,2, \ldots, N-1 .
\end{gather*}
$$

2. Transport Coefficients, Thermophysical and Thermokinetic Constants. The resultant homogeneous chemical reactions in a permeable layer of peat [11-13] are

$$
\begin{gather*}
\mathrm{CO}+\frac{1}{2} \mathrm{O}_{2}=\mathrm{CO}_{2}  \tag{12}\\
\mathrm{CH}_{4}+2 \mathrm{O}_{2}=\mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}
\end{gather*}
$$

The equations of chemical kinetics have the form [13]

$$
\begin{gathered}
\frac{d y_{1}}{d t}=-k_{1} x_{1} x_{3}^{0.25} T^{-2.25} \exp \left(-\frac{E_{1}}{R T}\right)=r_{1} \\
\frac{d y_{2}}{d t}=-k_{2} x_{5}^{-0.5} x_{3}^{1.5} \frac{P}{T} \exp \left(-\frac{E_{2}}{R T}\right)=r_{2}
\end{gathered}
$$

For evaporation of the bound water in the volume of the condensed phase of peat the Hertz-Knudsen law is used [11, 12]:

$$
R_{2 \mathrm{~s}}=\frac{s_{2} M_{2} A_{\mathrm{s}} \varphi_{2}\left[k_{2 \mathrm{~s}} \exp \left(-E_{2 \mathrm{~s}} / R T\right)-P_{2}\right]}{\left(2 \pi R T M_{2}\right)^{0.5}}
$$

In order to find the partial pressure of water vapor in a layer of peat $P_{2}$ the dalton law [14] is used, according to which $P_{2} / P=x_{2}$. Then for $P_{2}$ we have the expression

$$
P_{2}=P c_{2} \frac{M}{M_{2}}
$$

The effective diffusion coefficient is taken from the Fristrom-Westenberg formula [10, 15]:

$$
D_{\alpha}=\left(1-c_{\alpha}\right)\left(\sum_{\substack{j=1 \\ j \neq \alpha}}^{N} \frac{x_{j}}{d_{\alpha, j}}\right)^{-1}, d_{\alpha, j}=2.628 \cdot 10^{-7} \frac{\left[\left(M_{\alpha}+M_{j}\right) /\left(M_{\alpha} M_{j}\right)\right]^{0.5} T^{1.67}}{P \sigma_{j, \alpha}^{2}\left(\varepsilon_{j, \alpha} / k T\right)^{0.17}}
$$

The formula for the thermal conductivity of the gas phase $\lambda_{j}, j=1,2, \ldots, N$, was taken from [16]:

$$
\lambda_{5 \mathrm{~s}}=\sum_{i=1}^{N} \lambda_{i} c_{i}, \quad \lambda_{j}=\lambda_{j}^{0}\left(0.115+0.354 \frac{c_{p, j}}{R}\right), \quad \lambda_{i}^{0}=8.32 \cdot 10^{-2} \frac{\sqrt{M_{i} T}}{M_{i} \sigma_{i}^{2} \Omega^{(2.2)}} .
$$

The heat capacity coefficients of gas phase components $c_{p, j}=a_{j}+b_{j} T+c_{j} / T^{2}$ were taken from [17], and the values of $\lambda_{i \mathrm{~s}}$ in a condensed phase are given in [11]:

$$
c_{p 5}=\sum_{j=1}^{N} c_{p, j} c_{j}, \quad \lambda=\sum_{i=1}^{5} \lambda_{i s} \varphi_{i} .
$$

The sources and sinks from (3), (5)-(9) have the form

$$
R_{1}=\eta_{1} R_{1 \mathrm{~s}}-M_{1} r_{1}, \quad R_{2}=\eta_{2} R_{1 \mathrm{~s}}-R_{2 \mathrm{~s}}+2 M_{2} r_{2}, \quad R_{3}=-\eta_{3} R_{3 \mathrm{~s}}-M_{3} r_{1} / 2-2 M_{3} r_{2}
$$

$$
\begin{gathered}
R_{4}=M_{4}\left(r_{1}+r_{2}\right), \quad R_{5}=\eta_{4} R_{1 \mathrm{~s}}-M_{5} r_{2}, \quad R_{1 \mathrm{~s}}=k_{1 \mathrm{~s}} \rho_{1 \mathrm{~s}} \varphi_{1} \exp \left(-\frac{E_{1 \mathrm{~s}}}{R T}\right) \\
R_{3 \mathrm{~s}}=\frac{M_{\mathrm{c}}}{M_{3}} s_{3} k_{3 \mathrm{~s}} \varphi_{5} \rho_{5} \varphi_{3} c_{3} \exp \left(-\frac{E_{3 \mathrm{~s}}}{R T}\right), \quad Q=\left(1-\alpha_{\mathrm{c}}\right) R_{1 \mathrm{~s}}+R_{2 \mathrm{~s}}+R_{3 \mathrm{~s}} \\
\alpha_{\mathrm{c}}=\frac{M_{\mathrm{c} \infty}}{M_{\mathrm{in}}-M_{\mathrm{c} \infty}}, \quad \eta_{1}=\frac{M_{1}}{M_{\mathrm{in}}}, \quad \eta_{2}=\frac{M_{2}}{M_{\mathrm{in}}}, \quad \eta_{3}=\frac{M_{3}}{M_{\mathrm{c}}}, \quad \eta_{4}=\frac{M_{5}}{M_{\mathrm{in}}}
\end{gathered}
$$

3. Calculation Technique and Initial Data. The system of equations (3)-(8) with boundary-value conditions (9)-(11) was solved by the iteration-interpolation method of [18] with a variable step in time with a given accuracy. The program of numerical solution of the problem was tested by an exact analytical solution by the trial function method [18]. For different steps in space, the deviation of the numerical solution from the precise value over the given interval of time did not exceed $1 \%$.

The linear velocity of the peat smouldering surface (2) was determined from the formula

$$
\omega=\frac{\Delta z}{\Delta t_{*}}=\frac{z_{(k)}-z_{(k-1)}}{t_{*(k)}-t_{*(k-1)}} .
$$

For the thermophysical and thermokinetic parameters of peat the data of works [7,8,11, 19-21] were used. The thermophysical values of water and water vapor were taken from [22]. The results given below were obtained at $T_{\text {in }}=293 \mathrm{~K}, T_{*}=650 \mathrm{~K}, \omega_{*}=5 \cdot 10^{-6} \mathrm{~m} / \mathrm{sec}, P_{\text {in }}=1.013 \cdot 10^{5} \mathrm{~N} / \mathrm{m}^{2}, T_{\mathrm{e}}=700-900 \mathrm{~K}, \mu_{\text {in }}=1.81 \cdot 10^{-5} \mathrm{~kg} /(\mathrm{m} \cdot \mathrm{sec})$, $\alpha_{\mathrm{e}}=0.25-1.1 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right), M_{1}=28 \mathrm{~kg} / \mathrm{kmole}, M_{2}=18 \mathrm{~kg} / \mathrm{kmole}, M_{3}=32 \mathrm{~kg} / \mathrm{kmole}, M_{4}=44 \mathrm{~kg} / \mathrm{kmole}, M_{5}=16$ $\mathrm{kg} / \mathrm{kmole}, M_{6}=28 \mathrm{~kg} / \mathrm{kmole}, M_{\mathrm{c}}=12 \mathrm{~kg} / \mathrm{kmole}, M_{\mathrm{in}}=130 \mathrm{~kg} / \mathrm{kmole}, g=9.8 \mathrm{~m} / \mathrm{sec}^{2}, h=1 \mathrm{~m}, \rho_{1 \mathrm{~s}}=810-440$ $\mathrm{kg} / \mathrm{m}^{3}, \rho_{2 \mathrm{~s}}=10^{3} \mathrm{~kg} / \mathrm{m}^{3}, \rho_{3 \mathrm{~s}}=130 \mathrm{~kg} / \mathrm{m}^{3}, \rho_{4 \mathrm{~s}}=130 \mathrm{~kg} / \mathrm{m}^{3}, c_{1 \mathrm{~s}}=1.29 \cdot 10^{3} \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K}), c_{2 \mathrm{~s}}=4.19 \cdot 10^{3} \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K})$, $c_{3 \mathrm{~s}}=1.02 \cdot 10^{3} \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K}), c_{4 \mathrm{~s}}=1.02 \cdot 10^{3} \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K}), d_{\mathrm{p}}=10^{-6} \mathrm{~m}, R=8.314 \mathrm{~J} /(\mathrm{mole} \cdot \mathrm{K}), \lambda_{1 \mathrm{~s}}=1.67 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), \lambda_{2 \mathrm{~s}}=$ $0.601 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), \lambda_{3 \mathrm{~s}}=0.041 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), \lambda_{4 \mathrm{~s}}=0.041 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), A_{\mathrm{s}}=0.08, k_{1 \mathrm{~s}}=2 \cdot 10^{4} \mathrm{sec}^{-1}, E_{1 \mathrm{~s}}=54.47 \mathrm{~kJ} / \mathrm{mole}$, $q_{1 \mathrm{~s}}=-10^{3} \mathrm{~J} / \mathrm{kg}, k_{2 \mathrm{~s}}=10^{6} \mathrm{~kg} /\left(\mathrm{sec}^{2} \cdot \mathrm{~m}\right), E_{2 \mathrm{~s}}=16.76 \mathrm{~kJ} / \mathrm{mole}, q_{2 \mathrm{~s}}=10^{6} \mathrm{~J} / \mathrm{kg}, k_{3 \mathrm{~s}}=10^{5} \mathrm{~m} / \mathrm{sec}, E_{3 \mathrm{~s}}=41.87$ $\mathrm{kJ} / \mathrm{mole}, q_{3 \mathrm{~s}}=10^{5} \mathrm{~J} / \mathrm{kg}, c_{1 \text { in }}=0.05, c_{2 \text { in }}=510^{-5}, c_{3 \text { in }}=0.05-0.23, c_{4 \text { in }}=10^{-5}, c_{5 \text { in }}=0.2, c_{\alpha \mathrm{e}}=c_{\alpha \text { in }}, \alpha=1,2,4$, $5, c_{3 \mathrm{e}}=10^{-3}, \varphi_{1 \text { in }}=0.5-0.7, \varphi_{2 \text { in }}=0.05-0.4, \varphi_{3 \text { in }}=10^{-3}, \varphi_{4 \text { in }}=10^{-5}, s_{2}=0.08, s_{3}=0.05, \alpha_{4}=0.7, \eta_{1}=0.2$, $\eta_{2}=0.02, \eta_{3}=0.3$.
4. Analytical Solution of the Problem of Heating an Inert Layer of Peat and Checking of Numerical Solution. An exact analytical solution of the problem of heating a semi-infinite inert body under boundary conditions of the third kind:

$$
\begin{aligned}
& \frac{\partial T}{\partial t}=a \frac{\partial^{2} T}{\partial z^{2}}, \quad t>0, \quad 0<z<\infty,\left.\quad T\right|_{t=0}=T_{\mathrm{in}} \\
& \left.\lambda \frac{\partial T}{\partial z}\right|_{z=0}+\alpha_{\mathrm{e}}\left(T_{\mathrm{e}}-\left.T\right|_{z=0}\right)=0,\left.\quad T\right|_{z \rightarrow \infty}=T_{\mathrm{in}}
\end{aligned}
$$

was found in [23]:

$$
\begin{gathered}
T=T_{\mathrm{e}}-\left(T_{\mathrm{e}}-T_{\mathrm{in}}\right)\left[\Phi\left(\frac{z}{2 \sqrt{a t}}\right)+\exp (\gamma z+\psi) \Phi^{*}\left(\frac{z}{2 \sqrt{a t}}+\sqrt{\psi}\right)\right], \\
\Phi(z)=\frac{2}{\sqrt{\pi}} \int_{0}^{z} \exp \left(-y^{2}\right) d y, \Phi^{*}(z)=1-\Phi(z), \quad \gamma=\alpha_{\mathrm{e}} / \lambda, \quad \psi=a t \gamma^{2} .
\end{gathered}
$$

TABLE 1. Comparison of the Numerical and Analytical Solutions of the Problem of Heating of an Inert Body

| $\mathrm{t}, \min$ | $\boldsymbol{T}$ | $Z, \mathrm{~m}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 0.005 | 0.01 | 0.015 | 0.02 | 0.025 | 0.1 |
| 0.1 | $T_{\text {an }}$ | 294.98 | 293.46 | 293 | 293 | 293 | 293 | 293 |
|  | $T$ | 294.70 | 293.17 | 293.01 | 293 | 293 | 293 | 293 |
|  | $\left\|T-T_{\text {an }}\right\|$ | 0.28 | 0.29 | 0.01 | 0 | 0 | 0 | 0 |
|  | $T_{\text {an }}$ | 296.24 | 294.20 | 293.51 | 293 | 293 | 293 | 293 |
|  | $T$ | 296.36 | 293.76 | 293.12 | 293.01 | 293 | 293 | 293 |
|  | $\left\|T-T_{\text {an }}\right\|$ | 0.12 | 0.34 | 0.39 | 0.01 | 0 | 0 | 0 |
|  | $T_{\text {an }}$ | 297.58 | 294.68 | 293.36 | 293.03 | 293 | 293 | 293 |
|  | $T$ | 298.14 | 294.88 | 293.54 | 293.12 | 293 | 293 | 293 |
|  | $\left\|T-T_{\text {an }}\right\|$ | 0.56 | 0.20 | 0.22 | 0.08 | 0 | 0 | 0 |
|  | $T_{\text {an }}$ | 299.78 | 296.42 | 294.25 | 293.36 | 293.12 | 293 | 293 |
|  | $T$ | 300.48 | 296.82 | 294.68 | 293.64 | 293.21 | 293.05 | 293 |
|  | $\left\|T-T_{\text {an }}\right\|$ | 0.70 | 0.40 | 0.43 | 0.28 | 0.08 | 0.05 | 0 |

TABLE 2. Time of Ignition of Peat Samples ( $t_{*}, h$ )

| $\rho_{1 \mathrm{~s}}, \mathrm{~kg} / \mathrm{m}^{3}$ | $\varphi_{1 \text { in }}$ | $\varphi_{\text {2in }}$ | $\alpha_{\mathrm{e}}, \mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$ | $T_{\mathrm{e}}$, K |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 900 | 800 | 700 |
| 810 | 0.7 | 0.05 | 1.0 | 6.71 | 10.34 | 18.09 |
|  |  |  | 0.5 | 20.7 | 30.02 | 49.4 |
|  |  |  | 0.25 | 60.1 | 89.9 | 326 |
| 770 | 0.7 | 0.1 | 1.0 | 10.16 | 14.82 | 28.5 |
|  |  |  | 0.5 | 32 | 56.6 | 101 |
|  |  |  | 0.25 | 205 | 285 | - |
| 600 | 0.7 | 0.2 | 1.0 | 15.7 | 23.4 | 74.2 |
|  |  |  | 0.5 | 67.6 | 114 | 305 |
|  |  |  | 0.25 | 269 | - | - |
| 540 | 0.6 | 0.3 | 1.0 | 25.5 | 52.2 | 144 |
|  |  |  | 0.5 | 114 | 173 | 378 |
| 440 | 0.5 | 0.4 | 1.0 | 55 | 103 | 221 |
|  |  |  | 0.5 | 198 | 234 | - |

The results given in Table 1 were obtained at $T_{\mathrm{in}}=293 \mathrm{~K}, T_{\mathrm{e}}=893 \mathrm{~K}, \alpha_{\mathrm{e}}=2 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right),\left.\left(\sum_{i=1}^{4} c_{i s} \rho_{i \mathrm{~s}} \varphi_{i}\right)\right|_{t=0}=$ $1.378 \cdot 10^{6} \mathrm{~J} /\left(\mathrm{m}^{3} \cdot \mathrm{~K}\right),\left.\left(\sum_{i=1}^{4} \lambda_{i \mathrm{~S}} \varphi_{i}\right)\right|_{t=0}=1.301 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), a=0.9445 \cdot 10^{-6} \mathrm{~m}^{2} / \mathrm{sec}, \gamma=1.53728 \mathrm{~m}^{-1}, h=0.5 \mathrm{~m}$ ( $h$ models the infinity number). It is seen from the table that for small moments of time the error of numerical solution $T$ does not exceed the analytical $T_{\text {an }}$ with an accuracy less than $1 \%$.
5. Results of Numerical Solution and Their Analysis. First, we consider the regime of ignition and smouldering of a layer of peat under different environmental conditions (ignition from the ground combustion seat). The value $t=t_{*}$ is called the time of the occurrence of peat fire at which for $T \geq T_{*}$ the velocity of smouldering exceeds the characteristic velocity $\omega_{*}$, and the temperature of the reagent surface $T_{\mathrm{w}}$ increases sharply. For the sake of definiteness we assume that $T_{*}=650 \mathrm{~K}$ and $\omega_{*}=5 \cdot 10^{-6} \mathrm{~m} / \mathrm{sec}$ are known from experiment [2].


Fig. 1. Dependence of the time of ignition of peat on the heat transfer coefficient for an external source having temperature $T_{\mathrm{e}}=700 \mathrm{~K}$ (1), 800 K (2), 900 K (3). $t_{*}, \mathrm{~h} ; \alpha_{\mathrm{e}}, \mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$.
Fig. 2. The temperature of peat vs. the spatial variable at different time in-
stants: 1) $t=5$; 2) 10 ; 3) 15 ; 4) 20 ; 5) $22.16 \mathrm{~h} . T$, K; z, m.

Table 2 presents the time of ignition of a reagent at different values of $T_{\mathrm{e}}, \alpha_{\mathrm{e}}, \rho_{1 \mathrm{~s}}, \varphi_{1 \text { in }}, \varphi_{2 \text { in }}$ for $c_{3 \text { in }}=0.23$, values of density $\sum_{i=1}^{4} \rho_{i s} \varphi_{i i n}=620 \mathrm{~kg} / \mathrm{m}^{3}$, and reference input data from Item 3. As is seen from the table, as the quantity of moisture increases and the density of peat $\rho_{1 \mathrm{~s}}$ decreases (increase in the looseness of the sample), the time of ignition increases. This is due to the excess of heat removal by moisture evaporation over heat input from the exothermal reaction of coke oxidation and to the presence of air in the pores, which reduces the effective coefficient of heat conduction and increases the time of heating of a sample. As a result, at $T_{\mathrm{e}}=700 \mathrm{~K}, \alpha_{\mathrm{e}} \leq 0.25 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$; $\varphi_{2 \text { in }}=0.1 ; 700 \mathrm{~K} \leq T_{\mathrm{e}} \leq 800 \mathrm{~K}, \alpha_{\mathrm{e}} \leq 0.25 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right), \varphi_{2 \text { in }}=0.2 ; T_{\mathrm{e}}=700 \mathrm{~K}, \alpha_{\mathrm{e}} \leq 0.5 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right), \varphi_{2 \text { in }}=0.4$ and at large moisture content $\left(\varphi_{2 \mathrm{in}} \geq 0.3\right)$ for $700 \mathrm{~K} \leq T_{\mathrm{e}} \leq 900 \mathrm{~K}, \alpha_{\mathrm{e}} \leq 0.25 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$ there is no ignition of the reagent, and the velocity of smouldering of samples is comparable in order of magnitude with the rate of peat pyrolysis. This is due to the decrease in the intensity of the effect of the external combustion seat and to the increase in heat removal because of the increase in the consumption of energy on water evaporation.

In Fig. 1 the dependence of the time of reagent ignition on the heat transfer coefficient is presented for an external source at $\varphi_{1 \text { in }}=0.7, \varphi_{2 \text { in }}=0.2, \rho_{1 \mathrm{~s}}=600 \mathrm{~kg} / \mathrm{m}^{3}, c_{3 \text { in }}=0.23$ and the input data from Item 3. From the analysis of the numerical solution of the problem it follows that with increase in the intensity of external effect $\left(T_{\mathrm{e}}\right.$, $\alpha_{e}$ ) the time of ignition becomes shorter. With decrease in the value of the heat transfer coefficient to $\alpha_{e} \leq 0.25$ $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$ for $700 \mathrm{~K} \leq T_{\mathrm{e}} \leq 900 \mathrm{~K}$ the velocity of smouldering does not exceed the rate of peat pyrolysis and there is no ignition of the sample. This is associated with both the decrease in the mass exchange with the environment (insufficient amount of oxygen in the pores of the reagent) and with the increase in heat removal into the environment which becomes appreciable and exceeds the heat input from the exothermal reaction of coke oxidation. Such an effect of the absence of peat smouldering (see below) is observed if the initial concentration of oxygen in the pores of the reagent is small $\left(c_{3 \text { in }} \leq 0.05\right)$ at $T_{\mathrm{e}} \leq 800 \mathrm{~K}$ and $\alpha_{\mathrm{e}} \leq 1.1 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$.

Figure 2 depicts the temperature fields of the condensed phase, and Fig. 3 presents the concentrations of components in a permeable medium (peat) over the space for the mode of smouldering at different time instants at $\varphi_{\text {lin }}=$ $0.7, \varphi_{2 \text { in }}=0.2, c_{3 \text { in }}=0.23, T_{\mathrm{e}}=800 \mathrm{~K}, \alpha_{\mathrm{e}}=1.1 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$, and reference data from Item 3. The volumetric fractions of the components of the porous medium over the space are given in Fig. 4 for the initial reagent ( $\varphi_{1}$ ), bound water in a liquid-droplet state $\left(\varphi_{2}\right)$, coke $\left(\varphi_{3}\right)$, and soot $\left(\varphi_{4}\right)$. From the analysis of the numerical solution of the problem it follows that with an increase in the temperature of the permeable fragment of the medium first there occur heating and evaporation of the bound water, with the volumetric fraction of the bound water disappearing at $t \geq 10$ and converting into the concentration of $\mathrm{H}_{2} \mathrm{O}$ vapors (see Fig. 3a). In the region of high temperatures ( $T>380 \mathrm{~K}$ ) the



Fig. 3. Distribution of dimensionless concentrations of components $\left(\mathrm{CO}, \mathrm{H}_{2} \mathrm{O}\right.$, $\mathrm{O}_{2}, \mathrm{CO}_{2}, \mathrm{CH}_{4}, \mathrm{~N}_{2}$ ) over the depth of a sample for the reference data at the same time instants as in Fig. 2. $z, \mathrm{~m}$.


Fig. 4. Dimensionless volumetric fractions vs. the spatial variable at the same time instants as in Fig. 2. $z$, m.
Fig. 5. The linear velocity of peat smouldering vs. time (the initial data of Fig. 2). $\omega, \mathrm{m} / \mathrm{sec} ; t$, h.
process of pyrolysis of the initial reagent begins with appearance of the main mass of water vapors and coke (see lines $\varphi_{3}$ in Fig. 4). Thereafter, the pyrolysis product - coke - begins to burn out (smoulder) as a result of the exothermal reaction of oxidation into the depth of the fragment of the porous medium with the formation of soot (see the line $\varphi_{4}$ in Fig. 4 at $t=22.16 \mathrm{~h}$ ). At the same time, the pyrolysis products - gaseous CO, which appears at $T \leq$ 700 K , and methane - practically do not oxidize in the course of homogeneous reactions from (12) with the formation of $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ (see the lines $c_{4}$ in Fig. 3a and the lines $c_{5}$ in Fig. 3b) at $t=22.16 \mathrm{~h}$, since the concentration of the latter is small or changes insignificantly by virtue of the low temperatures of the reacting medium. It is known [13] that the gas-phase reactions of oxidation (12) begin intensely at $T>1000 \mathrm{~K}$.

It should be noted that during the entire time of heating and smouldering of peat, mainly in a permeable medium, the inert component nitrogen dominates (the curves $c_{6}$ in Fig. 3b). With time, when $t>5 \mathrm{~h}$, as a result of drying, pyrolysis of peat, and subsequent smouldering because of the decrease in the concentration of oxygen (the lines $c_{3}$ in Fig. 3a), the appearance of gaseous products water vapors, carbon oxide, and dioxide (the curves $c_{2}, c_{1}, c_{4}$, respectively, in Fig. 3a), somewhat displaces nitrogen in the permeable medium in the vicinity of $T_{\mathrm{w}}$, and the velocity of smouldering of peat is determined by the rate of supply of oxygen into the seat of combustion from the environment.

Figure 5 presents the value of the linear velocity of the process of smouldering of peat vs. time. The dependence $\omega=\omega(t)$ has a nonmonotonic character. First, the velocity of smouldering increases from zero to a certain high
value, which is due to the ignition from the external combustion seat; thereafter the velocity of smouldering decreases. The latter is attributed to the complete combustion of the heated layer of peat and to the formation of the zone of nonstationary smouldering, which represents the combination of the region of filtration that pulls ahead of the zone of chemical conversion and of the very zone of reaction [10]. After the formation of the zone of smouldering, the flame begins to propagate with a gradual acceleration, and a rapid increase in the temperature of the reagent surface occurs (see curve 5 in Fig. 2), with the average value of the velocity of peat smouldering being in agreement in order of magnitude with experimental data of [2, 7]. As the concentration of oxygen $c_{3 \text { in }}$ decreases from 0.23 to 0.11 and 0.05 , the time of ignition of the reagent increases up to $t_{* 1}=22.16, t_{* 2}=25.3$, and $t_{* 3}=45.7$, which is due to the limitation of the process of smouldering of the reagent by the content of the oxidant in the pores of the peat.

It is of interest to investigate the influence of the dependence of the content of the initial volumetric fractions of the initial reagent and moisture $\left(\varphi_{i i n}, i=1,2\right)$ on the rate of the process of smouldering of peat. As the volumetric fraction of moisture decreases from $\varphi_{2 \text { in }}=0.15$ to $\varphi_{2 \text { in }}=0.1$ for the reference value $\varphi_{1 \mathrm{in}}=0.7$, the time of ignition shortens from $t_{* 1} \leq 17.5$ to $t_{* 2} \leq 12.5$, which is due in the main to the expenditure of heat on evaporation of the bound water in the peat. With a decrease in the store of the initial value of the initial permeable reagent from $\varphi_{1 \mathrm{in}}=$ $0.7, \varphi_{1 \text { in }}=0.6$ to $\varphi_{1 \text { in }}=0.5$ for the reference value of moisture $\varphi_{2 \text { in }}=0.2$ the time of ignition increases: $t_{* 3} \leq 22.16$, $t_{* 4} \leq 33.7$, and $t_{* 5} \leq 42.6$, which is explained by the increase in the porosity of peat - the presence of air, which decreases the effective coefficient of heat conduction and increases the time of heating of samples. This result agrees qualitatively with the experimental data of [7].

## CONCLUSIONS

1. A new general statement of the problem of ignition of a layer of peat is given, including specific data with account for the processes of drying, pyrolysis, and oxidation of gaseous and condensed products.
2. As a particular case, the problem of heating a solid layer is solved with thermophysical constants in the absence of chemical reactions. The results of calculations coincided with the well-known analytical solution [23].
3. When $T_{\mathrm{e}}$ and $\alpha_{\mathrm{e}}$ change within $700 \leq T_{\mathrm{e}} \leq 900 \mathrm{~K}$ and $0.25 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right) \leq \alpha_{\mathrm{e}} \leq 1.1 \mathrm{~W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right)$, the period of ignition and further smouldering of peat are limited by the intensity of the external seat of combustion, by the initial content of the oxidant in the pores of the reagent, as well as by the processes of drying, pyrolysis, and exothermal reaction of oxidation of the coked layer.
4. The results of calculations of the velocity of peat smouldering agree with the experimental data of [2, 7].

## NOTATION

a, thermal diffusivity, $\mathrm{m}^{2} / \mathrm{sec} ; A_{\mathrm{s}}$, accommodation coefficient; $c$, mass concentration of components; $c_{p}$, heat capacity coefficient, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K}) ; D$, diffusion coefficient, $\mathrm{m}^{2} / \mathrm{sec} ; d_{\mathrm{p}}$, diameter of cylindrical pores, $\mathrm{m} ; E_{i}, i=1$, 2 , energy of activation of homogeneous reactions of oxidation, $\mathrm{J} / \mathrm{mole} ; g$, free fall acceleration, $\mathrm{m} / \mathrm{sec}^{2} ; h$, thickness of the initial permeable layer of peat, $\mathrm{m} ; k$, Boltzmann constant, $\mathrm{J} / \mathrm{K} ; k_{i}, i=1,2$, pre-exponential factors of oxidation reactions, $\sec ^{-1} ; M$, molecular weight, $\mathrm{kg} / \mathrm{kmole} ; P$, gas pressure in pores, $\mathrm{N} / \mathrm{m}^{2} ; q_{i}, i=1,2$, thermal effects of oxidation reactions, $\mathrm{J} / \mathrm{kg} ; q_{i s}, i=1,4$, thermal effect, $\mathrm{J} / \mathrm{kg} ; R$, universal gas constant, $\mathrm{J} /($ mole $\cdot \mathrm{K}) ; r_{1}$ and $r_{2}$, molar-volumetric rates of oxidation of carbon oxide and methane; $R_{1 \mathrm{~s}}$, mass rate of decomposition of a dry reagent (peat), $\mathrm{kg} /\left(\mathrm{m}^{3} \cdot \mathrm{sec}\right) ; R_{2 \mathrm{~s}}$, mass rate of evaporation of bound water in peat, $\mathrm{kg} /\left(\mathrm{m}^{3} \cdot \mathrm{sec}\right) ; R_{3 \mathrm{~s}}$, mass rate of formation of coke breeze, $\mathrm{kg} /\left(\mathrm{m}^{3} \cdot \mathrm{sec}\right) ; R_{i}, i=\overline{1,4}$, mass rate of formation and disappearance of the components of the gas phase in the diffusion equation, $\mathrm{kg} /\left(\mathrm{m}^{3} \cdot \mathrm{sec}\right) ; s_{*}(t)$, interface between the gaseous and condensed phases, $\mathrm{m} ; s_{2}$, specific surface of water evaporation, $\mathrm{m}^{-1} ; s_{3}$, specific surface of the reaction of carbon, $\mathrm{m}^{-1} ; t$, time, $\mathrm{h} ; t_{*(k)}$ and $t_{*(k-1)}$, time in which the smouldering temperature $T_{*}$ is attained at $z=z_{(k)}$ and $z=z_{(k-1)} ;(k-1)$, the previous layer over $z ; T_{i}$, temperature, $\mathrm{K} ; v$, velocity of the gaseous products of the reaction of decomposition of peat, $\mathrm{m} / \mathrm{sec} ; x_{j}=c_{j} M / M_{j}, j=1, \ldots, 5$, molar concentration; $y=\rho c / M$, molar-volumetric concentration, mole $/ \mathrm{cm}^{3} ; z$, space coordinate, $\mathrm{m} ; \alpha_{\mathrm{e}}$, heat transfer coefficient, $\mathrm{W} /\left(\mathrm{m}^{2} \cdot \mathrm{~K}\right) ; \alpha_{4}=v_{4} M_{4 \mathrm{~s}} /\left(v_{3} M_{3 \mathrm{~s}}\right)$, reduced stoichiometric coefficient [10]; $\alpha_{\mathrm{c}}$, fraction of coke in the course of reaction of peat pyrolysis; $\beta_{\mathrm{e}}$, mass transfer coefficient, $\mathrm{kg} /\left(\mathrm{m}^{2} \cdot \mathrm{sec}\right)$; $\varepsilon_{j, \alpha}$, potential energy of interaction of molecules, $\mathrm{J} ; \eta_{i}, i=\overline{1,4}$, dimensionless parameters; $\lambda$, thermal conductivity, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K}) ; \mu=\mu_{\mathrm{in}}\left(T / T_{\mathrm{in}}\right)^{0.5}$, coefficient of dy-
namic viscosity of a mixture of gases, $\mathrm{kg} /(\mathrm{m} \cdot \mathrm{sec}) ; \xi=\xi_{*} \varphi_{5}^{3} /\left(1-\varphi_{5}\right)^{2}$, function describing the influence of the volumetric fraction of a gas on resistance, $\xi_{*}=d_{\mathrm{p}}^{2} / 120$, characteristic permeability, $\mathrm{m}^{2} ; \sigma_{i, j}$, section of interaction of molecules, $\AA ; \rho$, density, $\mathrm{kg} / \mathrm{m}^{3} ; \rho_{5}$, gas phase density, $\mathrm{kg} / \mathrm{m}^{3} ; \varphi$, dimensionless volumetric fraction; $\omega$, linear velocity of peat smouldering, $\mathrm{m} / \mathrm{sec}$. Subscripts: s , condensed phase; e, environment; c, coke; in, initial value; 1, ..., 6, in a gas phase corresponds to carbon oxide, water vapors, oxygen, carbon dioxide, methane, nitrogen; $1 \mathrm{~s}, \ldots, 4 \mathrm{~s}$, in a condensed phase - peat, bound water, coke, soot; p, pore; an , analytical solution. Superscript: *, characteristic value.

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